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Lesson 8-7 Factoring Special Cases

Reminder: DIFFERENCE OF SQUARES: $a^2 - b^2 = (a + b)(a - b)$

A. $16y^2 - 81z^2 = (4y)^2 - (9z)^2$

$= (4y + 9z)(4y - 9z)$

B. $3b^3 - 27b = 3b(b^2 - 9)$ (Factor out the common factor.)

$= 3b(b^2 - 3^2)$

$= 3b(b+3)(b-3)$

Apply a Factoring Technique More Than Once

C. $4y^4 - 2500 = 4(y^4 - 625)$

$= 4((y^2)^2 - 25^2)$

$= 4(y^2+25)(y^2-25)$

$= 4(y^2+25)(y^2-5^2)$

$= 4(y^2+25)(y+5)(y-5)$

D. $3y^4 - 48 = 3(y^4 - 16)$

$= 3((y^2)^2 - 4^2)$

$= 3(y^2+4)(y^2-4)$

$= 3(y^2+4)(y^2-2^2)$

$= 3(y^2+4)(y+2)(y-2)$

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Lesson 8-7 Factoring Special Cases

Perfect Square Trinomials

E.

1st term must be a perfect square	Middle terms must be twice the product of the square roots of the 1st and last terms.	Last term must be a perfect square
$25x^2 = (5x)^2$	$2(5x)(3) = 30x$	$9 = (3)^2$

$25x^2 - 30x + 9 =$ $(5x-3)^2$

F. $49y^2 + 42y + 36 =$ not a perfect squared trinomial

$(7y)^2 + 2(7y)(6) + 6^2$

$\underbrace{2(7y)(6)}_{84y}$

the middle term is not $2 \cdot a \cdot b$

G. $6x^2 - 96 =$ $6(x^2 - 16)$

$6(x+4)(x-4)$

H. $16y^2 + 8y - 15 =$ $(y^2 + 8y - 240) =$

$(y + \frac{20}{16})(y - \frac{12}{16}) = (y + 5/4)(y - 3/4) = (4y+5)(4y-3)$

$16 \cdot 15 = -240$
$24, -10$
$-12, 20$

Factoring Polynomials - Summary			
Number of Terms	Factoring Technique		Example
2 or more	greatest common factor		$3x^3 + 6x^2 - 15 = 3x(x^2 + 2x - 5)$
2	Difference of squares	$a^2 - b^2 = (a + b)(a - b)$	$4x^2 - 25 = (2x + 5)(2x - 5)$
3	Perfect square trinomial	$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$X^2 + 6x + 9 = (x + 3)^2$ $4x^2 - 4x + 1 = (2x + 1)^2$
	$x^2 + bx + c$	$x^2 + bx + c = (x + m)(x - n)$ when $m + n = b$ and $mn = c$	$X^2 - 9x + 20 = (x - 5)(x - 4)$
	$ax^2 + bx + c$	$ax^2 + bx + c = ax^2 + mx + nx + c$ whem $m + n = b$ and $mn = ac$. Then use factoring by grouping	$6x^2 - x - 2 = 6x^2 + 3x - 4x - 2$ $= 3x(2x + 1) - 2(2x + 1)$ $= (2x + 1)(3x - 2)$
4 or more	Factor by grouping	$ax + bx + ay + by$ $= x(a + b) + y(a + b)$ $= (a + b)(x + y)$	$3xy - 6y + 5x - 10$ $= (3xy - 6y) + (5x - 10)$ $= 3y(x - 2) + 5(x - 2)$ $= (x - 2)(3y + 5)$

Lesson 8-7 Factoring Special Cases

Practice:

Factor each polynomial if possible. If the polynomial cannot be factored, write *prime*.

1. $98x^2 - 200y^2$

$2(49x^2 - 100y^2)$

$2(7x + 10y)(7x - 10y)$

2. $X^2 + 22x + 121$

$x^2 \quad 11^2$

$(x + 11)^2$

3. $81 + 18s + s^2$

$s^2 + 18s + 81$

$(s+9)^2$

4. $25c^2 - 10c - 1$

$5c \quad 1$

prime

5. $169 - 26r + r^2$

$r^2 - 26r + 169$

$(r - 13)^2$

6. $7x^2 - 9x + 2$

$x^2 - 9x + 14$

$(x - \frac{7}{7})(x - \frac{2}{7})$

$(x - 1)(7x - 2)$

7. $16m^2 + 48m + 36$

$(4m)^2 \quad 6^2$

$(4m + 6)^2$

8. $16 - 25a^2$

$(4)^2 \quad (5a)^2$

$(4 - 5a)(4 + 5a)$

9 $b^2 - 16b + 256$

$b \quad 16$

prime

10. $36x^2 - 12x + 1$

$(6x)^2 \quad 1^2$

$(6x - 1)^2$

11. $16a^2 - 40ab + 25b^2$

$(4a)^2 \quad (5b)^2$

$(4a - 5b)^2$

12. $8m^3 - 64m = 8m(m^2 - 8)$

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Lesson 8-7 Factoring Special Cases

Solving Equations With Perfect Squares

Perfect square trinomials will always have a ~~duplicate~~ repeated factor, so they will always have only 1 solution.

E. $4x^2 + 36x + 81 = 0$

$(2x)^2$ 9^2

$(2x+9)^2 = 0$

$2x + 9 = 0$

$2x = -9$

$x = -9/2$

Solution set is $\{-9/2\}$

F. $x^2 - 36 = 0$

$(x+6)(x-6) = 0$

$x+6=0$ or $x-6=0$

Solution set is $\{-6, 6\}$

Square Root Property

For any number n> 0, if $x^2 = n$, then $x = \pm\sqrt{n}$

Example: $x^2 = 9$

$x = \pm\sqrt{9}$ or ± 3

G. $(b-7)^2 = 36$

$\sqrt{(b-7)^2} = \sqrt{36}$ (square root property)

$b-7 = \pm 6$ (simplify)

$b-7=6$ or $b-7=-6$

$+7 \ \ +7$ $\quad$ $+7 \ \ +7$ (separate into two equations)

$b = 13$ $\quad$ $b = 1$ (simplify)

Solution set is $\{13, 1\}$

H. $(x-3)^2 = 25$

$\sqrt{(x-3)^2} = \sqrt{25}$

$x-3 = \pm 5$

$x-3=5$ $\quad$ $x-3=-5$

$+3 \ \ +3$ $\quad$ $+3 = +3$

$x = 8$ $\quad$ $x = -2$

$\{8, -2\}$

I. $y^2 + 12y + 36 = 100$

y^2 $2 \cdot y \cdot 6$ 6^2 10^2

$(y+6)^2 = 10^2$

$\sqrt{(y+6)^2} = \sqrt{10^2}$

$y+6 = \pm 10$

$y+6 = 10$ or $y+6 = -10$

$-6 \ \ -6$ $\quad$ $-6 \ \ -6$

$y = 4$ $\quad$ $y = -16$

$\{4, -16\}$